FRW Cosmology with Variable G and Λ

Mubasher Jamil[1,*] and Ujjal Debnath[2,†]

[1]*Center for Advanced Mathematics and Physics,*

National University of Sciences and Technology, H-12, Islamabad, Pakistan.

[2]*Department of Mathematics, Bengal Engineering and*

Science University, Shibpur, Howrah-711 103, India.

Abstract

We have considered a cosmological model of the FRW universe with variable G and Λ. The solutions have been obtained for flat model with particular form of cosmological constant. The cosmological parameters have also been obtained for dust, radiation and stiff matter. The statefinder parameters are analyzed and have shown that these depends only on w and ϵ. Further the lookback time, proper distance, luminosity distance and angular diameter distance have also been calculated for our model.

Keywords: Newton's gravitational constant; cosmological constant; cosmology.

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*Electronic address: mjamil@camp.nust.edu.pk

†Electronic address: ujjaldebnath@yahoo.com , ujjal@iucaa.ernet.in

I. INTRODUCTION

The Einstein field equation has two parameters - the gravitational constant G and the cosmological constant Λ. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry and matter in the Einstein field equations. In an evolving Universe, it appears natural to look at this "constant" as a function of time. Numerous suggestions based on different arguments have been proposed in the past few decades in which G varies with time [1, 2]. Dirac [3] proposed a theory with variable G motivated by the occurrence of large numbers discovered by Weyl, Eddington and Dirac himself. Many other extensions of Einstein's theory with time dependent G have also been proposed in order to achieve a possible unification of gravitation and elementary particle physics or to incorporate Mach's principle in general relativity [4]. From the point of view of incorporating particle physics into Einstein's theory of gravitation, the simplest approach is to interpret the cosmological constant Λ in terms of quantum mechanics and the Physics of vacuum [5]. The Λ term has also been interpreted in terms of the Higgs scalar field [6].

Λ as a function of time has also been considered by several authors in various variable G theories in different contexts [7]. With this in view, several authors [8, 9] have proposed linking the variation of G with that of Λ in the framework of general relativity. By considering the conservation of the energy-momentum tensor of matter and vacuum take together, many authors have invoked the idea of a decreasing vacuum energy and hence a varying cosmological constant Λ with cosmic expansion in the frame work of Einstein's theory. Present-day astronomical observations indicate [10] that the cosmological constant Λ is extremely negligible being $\leq 10^{-56}$ cm^2. But the value of this constant should be 10^{50} times larger according to the Glashow-Weinberg-Salam model [11] for electro-weak unification and 10^{107} times larger according to GUT [12] for grand unification.

In attempt to modify the general theory of relativity, Al-Rawaf and Taha [13] related the cosmological constant to the Ricci scalar R. This is written as a built-in-cosmological constant, i.e., $\Lambda \propto R$. Since the Ricci scalar contains a term of the form $\frac{\ddot{a}}{a}$, one adopts this variation for Λ i.e., $\Lambda \propto \frac{\ddot{a}}{a}$ [14]. Similarly, another two forms for Λ have been chosen as: $\Lambda \propto \rho$ and $\Lambda \propto \frac{\dot{a}^2}{a^2}$ [15]; where ρ is the energy density.

II. THE MODEL

The action of our model is [16]

$$S = \int d^4x \mathcal{L} = \int d^4x \Big\{ \sqrt{-g}\Big[\frac{R}{G} + F(G)\Big] + \mathcal{L}_m \Big\}, \tag{1}$$

where G is the Newton's gravitational constant and $F(G)$ is an arbitrary function of G while $\mathcal{L}_m$ is a matter Lagrangian. From the Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial G} = \nabla_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu G)}, \tag{2}$$

to obtain

$$\frac{\partial F}{\partial G} = \frac{R}{G^2}. \tag{3}$$

Moreover from the variation with respect to $g_{\mu\nu}$, we have from (1),

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} + g_{\mu\nu}\Big(\frac{1}{2}GF(G)\Big). \tag{4}$$

Writing

$$\frac{1}{2}GF(G) = \Lambda(t), \tag{5}$$

we get from (4)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} + g_{\mu\nu}\Lambda(t). \tag{6}$$

As a background geometry, we consider a spatially homogeneous and isotropic FRW line element

$$ds^2 = dt^2 - a^2(t)\Big[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big], \tag{7}$$

where $a(t)$ is the scale factor and $k = -1, 0, +1$ is the curvature parameter for spatially open, flat and closed universes, respectively.

We assume that the cosmic matter is represented by the energy-momentum tensor of a perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}. \tag{8}$$

Here ρ and p are the energy density and pressure of the cosmic matter, u_μ is the four velocity satisfying $u_\mu u^\mu = 1$.

We take the equation of state

$$p = (w - 1)\rho, \quad 1 \le w \le 2. \tag{9}$$

Now the for the metric (7) and stress-energy tensor (8), the field equations (6) take the form

$$-2\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 - \frac{k}{a^2} + \Lambda = 8\pi G p, \tag{10}$$

$$3\left(\frac{\dot{a}}{a}\right)^2 + 3\frac{k}{a^2} - \Lambda = 8\pi G \rho. \tag{11}$$

In view of vanishing of divergence of Einstein tensor, we have

$$8\pi G\left(\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a}\right) + 8\pi\rho\dot{G} + \dot{\Lambda} = 0. \tag{12}$$

The usual energy conservation equation $T^{\mu\nu}_{;\nu} = 0$ yields

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0. \tag{13}$$

Using (13) in (12) gives

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0. \tag{14}$$

Note that (14) implies that G is constant whenever Λ is constant and vice-versa. Making use of (9) in (14), we obtain

$$\rho = \frac{C_1}{a^{3w}}, \tag{15}$$

where C_1 is a constant of integration. We can determine the value of C_1 by assuming $w = w_0$, $\rho = \rho_c$ due to spatial flatness (ρ_c being the critical density) at $t = t_0$, where subscript '0' corresponds to the present value. It turns out $C_1 = \rho_c a_0^{3w_0}$. Thus (15) becomes

$$\rho = \rho_c \frac{a_0^{3w_0}}{a^{3w}}, \tag{16}$$

For spatially flat case $k = 0$, (10) and (11) become

$$8\pi G p = H^2(2q - 1) + \Lambda, \tag{17}$$

$$8\pi G \rho = 3H^2 - \Lambda, \tag{18}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, $q = -1 - \frac{\dot{H}}{H^2}$ is the deceleration parameter and $\Theta = 3H$ is the expansion scalar.

Sahni et al [17] introduced a pair of cosmological diagnostic pair $\{r, s\}$ which they termed as Statefinder. The two parameters are dimensionless and are geometrical since they are

derived from the cosmic scale factor alone, though one can rewrite them in terms of the parameters of dark energy and matter. Additionally, the pair gives information about dark energy in a model independent way i.e. it categorizes dark energy in the context of background geometry only which is not dependent on the theory of gravity. Hence geometrical variables are universal. Also this pair generalizes the well-known geometrical parameters like the Hubble parameter and the deceleration parameter. This pair is algebraically related to the equation of state of dark energy and its first time derivative.

The statefinder parameters were introduced to characterize primarily flat universe ($k = 0$) models with cold dark matter (dust) and dark energy. They were defined as

$$r \equiv \frac{\dddot{a}}{aH^3}, \tag{19}$$

$$s \equiv \frac{r - 1}{3(q - \frac{1}{2})}. \tag{20}$$

For cosmological constant with a fixed equation of state ($w = -1$) and a fixed Newton's gravitational constant, we have $\{1, 0\}$. Moreover $\{1, 1\}$ represents the standard cold dark matter model containing no radiation while Einstein static universe corresponds to $\{\infty, -\infty\}$ [18]. In literature, the diagnostic pair is analyzed for various dark energy candidates including holographic dark energy [19], agegraphic dark energy [20], quintessence [21], dilaton dark energy [22], Yang-Mills dark energy [23], viscous dark energy [24], interacting dark energy [25], tachyon [26], modified Chaplygin gas [27] and $f(R)$ gravity [28] to name a few.

III. SOLUTIONS OF DYNAMICAL EQUATIONS

We here assume the ansatz [29] for the variable cosmological constant

$$\Lambda(t) = \epsilon H^2, \tag{21}$$

where ϵ is a constant. Making use of (9),(17),(18) and (21), we obtain a differential equation

$$2\dot{H} + (3 - \epsilon)wH^2 = 0. \tag{22}$$

Solving (22), we get

$$H(t) = \frac{-2}{(\epsilon - 3)wt + 2C_2}, \tag{23}$$

where C_2 is an integration of constant. Note that C_2 can be determined like before to get

$$H(t) = \frac{2H_0}{H_0(3-\epsilon)(wt-w_0t_0)+2}.$$

(24)

Solving for t we get

$$t = \frac{2}{w(\epsilon-3)H_0}\left(1-\frac{H_0}{H}\right)+t_0\frac{w_0}{w}.$$

(25)

From (18) and (21), we get

$$\rho = \frac{1}{8\pi G}(3-\epsilon)H^2.$$

(26)

Integrating (24), we get

$$a(t) = C_3[(3-\epsilon)(tw-t_0w_0)H_0+2]^{\frac{2}{w(3-\epsilon)}}.$$

(27)

Using (27) in (16), we get

$$\rho(t) = \rho_c a_0^{3w_0} C_3^{-3w}[(3-\epsilon)(tw-t_0w_0)H_0+2]^{\frac{6}{(\epsilon-3)}}.$$

(28)

Using (27) in (21), we get

$$\Lambda(t) = \frac{4\epsilon H_0^2}{[H_0(3-\epsilon)(tw-t_0w_0)+2]^2}.$$

(29)

Making use of (29) and (28) in (14) to get

$$G(t) = \frac{H_0^2(3-\epsilon)[(3-\epsilon)(tw-t_0w_0)H_0+2]^{\frac{2\epsilon}{3-\epsilon}}}{2\pi\rho_c a_0^{3w_0} C_3^{-3w}}.$$

(30)

The deceleration parameter becomes

$$q = -1 + \frac{w(3-\epsilon)}{2}.$$

(31)

For an expanding universe, $q \leq -1$ which constrains $\epsilon < 3$. The expansion scalar takes the form

$$\Theta = \frac{6H_0}{H_0(3-\epsilon)(wt-w_0t_0)+2}.$$

(32)

For flat universe, the density parameter can be obtained as

$$\Omega = \frac{8\pi G\rho}{3H^2} = 1 - \frac{\epsilon}{3}$$

(33)

We now calculate the statefinder parameters $\{r,s\}$ as well

$$r = \frac{\dddot{a}}{aH^3}, \quad s = \frac{r-1}{3(q-\frac{1}{2})}.$$

(34)

The parameters take the form

$$r = \frac{1}{2}[1+w(\epsilon-3)][2+w(\epsilon-3)], \quad s = \frac{w}{3}(3-\epsilon).$$

(35)

Note that for $\epsilon = 3$, $\{r,s\}=\{1,0\}$ representing a static cosmological constant.

A.   Universe containing only dust

The matter (baryonic and non-baryonic) satisfies the EoS parameter $w = 1$. Thus cosmological parameters take the form

$$a(t) = C_3[(3 - \epsilon)(t - t_0 w_0)H_0 + 2]^{\frac{2}{3-\epsilon}}, \tag{36}$$

$$\rho(t) = \rho_c a_0^{3w_0} C_3^{-3}[(3 - \epsilon)(t - t_0 w_0)H_0 + 2]^{\frac{6}{\epsilon-3}}, \tag{37}$$

$$\Lambda(t) = \frac{4\epsilon H_0^2}{[H_0(3 - \epsilon)(t - t_0 w_0) + 2]^2}, \tag{38}$$

$$G(t) = \frac{H_0^2(3 - \epsilon)[(3 - \epsilon)(t - t_0 w_0)H_0 + 2]^{\frac{2\epsilon}{3-\epsilon}}}{2\pi \rho_c a_0^{3w_0} C_3^{-3}}, \tag{39}$$

$$q = \frac{1 - \epsilon}{2}, \tag{40}$$

$$\Theta = \frac{6H_0}{H_0(3 - \epsilon)(t - w_0 t_0) + 2}, \tag{41}$$

$$r = \frac{1}{2}(\epsilon - 2)(\epsilon - 1), \tag{42}$$

$$s = \frac{1}{3}(3 - \epsilon). \tag{43}$$

B.   Universe containing only radiation

The radiation satisfies the EoS parameter $w = 4/3$. Thus cosmological parameters take the form

$$a(t) = C_3\left[(3 - \epsilon)\left(\frac{4}{3}t - t_0 w_0\right)H_0 + 2\right]^{\frac{3}{2(3-\epsilon)}}, \tag{44}$$

$$\rho(t) = \rho_c a_0^{3w_0} C_3^{-4}\left[(3 - \epsilon)\left(\frac{4}{3}t - t_0 w_0\right)H_0 + 2\right]^{\frac{6}{(\epsilon-3)}}, \tag{45}$$

$$\Lambda(t) = \frac{4\epsilon H_0^2}{\left[(3 - \epsilon)\left(\frac{4}{3}t - t_0 w_0\right)H_0 + 2\right]^2}, \tag{46}$$

$$G(t) = \frac{H_0^2(3 - \epsilon)\left[(3 - \epsilon)\left(\frac{4}{3}t - t_0 w_0\right)H_0 + 2\right]^{\frac{2\epsilon}{3-\epsilon}}}{2\pi \rho_c a_0^{3w_0} C_3^{-4}}, \tag{47}$$

$$q = 1 - \frac{2\epsilon}{3}, \tag{48}$$

$$\Theta = \frac{6H_0}{(3-\epsilon)\left(\frac{4}{3}t - t_0 w_0\right)H_0 + 2}, \tag{49}$$

$$r = \frac{1}{9}(4\epsilon - 9)(2\epsilon - 3), \tag{50}$$

$$s = \frac{4}{9}(3 - \epsilon). \tag{51}$$

C. Universe containing only stiff matter

The stiff fluid satisfies the EoS parameter $w = 2$. Thus cosmological parameters take the form

$$a(t) = C_3[(3-\epsilon)(2t - t_0 w_0)H_0 + 2]^{\frac{1}{3-\epsilon}}, \tag{52}$$

$$\rho(t) = \rho_c a_0^{3w_0} C_3^{-6}[(3-\epsilon)(2t - t_0 w_0)H_0 + 2]^{\frac{6}{\epsilon-3}}, \tag{53}$$

$$\Lambda(t) = \frac{4\epsilon H_0^2}{[H_0(3-\epsilon)(2t - t_0 w_0) + 2]^2}, \tag{54}$$

$$G(t) = \frac{H_0^2(3-\epsilon)[(3-\epsilon)(2t - t_0 w_0)H_0 + 2]^{\frac{2\epsilon}{3-\epsilon}}}{2\pi \rho_c a_0^{3w_0} C_3^{-6}}, \tag{55}$$

$$q = 2 - \epsilon, \tag{56}$$

$$\Theta = \frac{6H_0}{H_0(3-\epsilon)(2t - w_0 t_0) + 2}, \tag{57}$$

$$r = (2\epsilon - 5)(\epsilon - 2), \tag{58}$$

$$s = \frac{2}{3}(3 - \epsilon). \tag{59}$$

IV. SOME CONSEQUENCES

In this section, we'll discuss lookback time, proper distance, luminosity distance and angular diameter as the following [30].

A. Lookback time

If a photon emitted by a source at the instant t and received at the time t_0 then the photon travel time or the lookback time $t - t_0$ is defined by

$$t - t_0 = \int_{a_0}^{a} \frac{da}{\dot{a}}, \tag{60}$$

where a_0 is the present value of the scale factor of the universe and can be obtained from (27) as (at $t = t_0$, $w = w_0$)

$$a_0 = 2^{\frac{2}{w_0(3-\epsilon)}} \, C_3. \tag{61}$$

The redshift z can be defined by

$$1 + z = \frac{a_0}{a}, \tag{62}$$

which simplifies to

$$t - t_0 = \frac{2}{w(\epsilon - 3)H_0} + \left(\frac{w_0}{w} - 1\right) t_0 + \frac{2^{\frac{w_0}{w}}}{w(3 - \epsilon)H_0}(1 + z)^{-\frac{w(3-\epsilon)}{2}}. \tag{63}$$

Since, $w > 0$ and $\epsilon < 3$ always, in our assumption. So for early universe i.e., for $z \to \infty$ we get

$$t - t_0 \approx \frac{2}{w(\epsilon - 3)H_0} + \left(\frac{w_0}{w} - 1\right) t_0, \tag{64}$$

and for late universe i.e., for $z \to -1$ we get

$$t - t_0 \approx \frac{2^{\frac{w_0}{w}}}{w(3 - \epsilon)H_0}(1 + z)^{-\frac{w(3-\epsilon)}{2}}. \tag{65}$$

B. Proper distance

If a photon emitted by a source and received by an observer at time t_0 then the proper distance between them is defined by

$$d = a_0 \int_a^{a_0} \frac{da}{a\dot{a}} = a_0 \int_t^{t_0} \frac{dt}{a}, \tag{66}$$

which simplifies to

$$d = \frac{2^{\frac{w}{w_0}} (1 + z)^{1 + \frac{w(\epsilon - 3)}{2}}}{(2 + w(\epsilon - 3))H_0} - \frac{2}{(2 + w_0(\epsilon - 3))H_0}. \tag{67}$$

C. Luminosity distance

If L be the total energy emitted by the source per unit time and ℓ be the apparent luminosity of the object then the luminosity distance is defined by

$$d_L = \left(\frac{L}{4\pi\ell}\right)^{\frac{1}{2}}. \tag{68}$$

Now for our model,

$$d_L = d(1 + z) = \frac{2^{\frac{w}{w_0}} (1 + z)^{2 + \frac{w(\epsilon - 3)}{2}}}{(2 + w(\epsilon - 3))H_0} - \frac{2(1 + z)}{(2 + w_0(\epsilon - 3))H_0}. \tag{69}$$

D.   Angular diameter

The angular diameter of a light source of proper distance D observed at t_0 is defined by

$$\delta = \frac{D(1+z)^2}{d_L}. \tag{70}$$

The angular diameter distance (d_A) is defined as the ratio of the source diameter to its angular diameter as

$$d_A = \frac{D}{\delta} = d_L(1+z)^{-2}, \tag{71}$$

which is simplifies to

$$d_A = \frac{2^{\frac{w}{w_0}}(1+z)^{\frac{w(\epsilon-3)}{2}}}{(2+w(\epsilon-3))H_0} - \frac{2(1+z)^{-1}}{(2+w_0(\epsilon-3))H_0}. \tag{72}$$

The angular diameter distance is maximum at

$$z_{max} = \left(\frac{2^{2-\frac{w}{w_0}}}{w(3-\epsilon)} \frac{2+w(\epsilon-3)}{2+w_0(\epsilon-3)} \right)^{\frac{2}{2+w(\epsilon-3)}} - 1, \tag{73}$$

and hence the maximum angular diameter distance will be

$$(d_A)_{max} = \frac{2}{H_0} \left(2^{2-\frac{w}{w_0}} \right)^{-\frac{2}{2+w(\epsilon-3)}} \left(\frac{2+w(\epsilon-3)}{w(3-\epsilon)(2+w_0(\epsilon-3))} \right)^{\frac{w(\epsilon-3)}{2+w(\epsilon-3)}}. \tag{74}$$

V.   CONCLUSIONS

In this work, we have considered a cosmological model of the homogeneous and isotropic FRW universe with variable Newton's gravitational constant and cosmological constant. The solutions have been obtained for flat model with barotropic fluid and some particular form of cosmological constant (i.e., $\Lambda = \epsilon H^2$). The cosmological parameters and deceleration parameter have been obtained for dust, radiation and stiff perfect fluid. The statefinder parameters are analyzed for these three types of fluids and have shown that these depends only on w and ϵ. From figure 1, we see that q decreases with w decreases and ϵ increases. Also figures 1-3 describe the natures of statefinder parameters for these three types of fluids. Further the lookback time, proper distance, luminosity distance and angular diameter distance have also been investigated. The maximum angular diameter

distance has also been found in our model.

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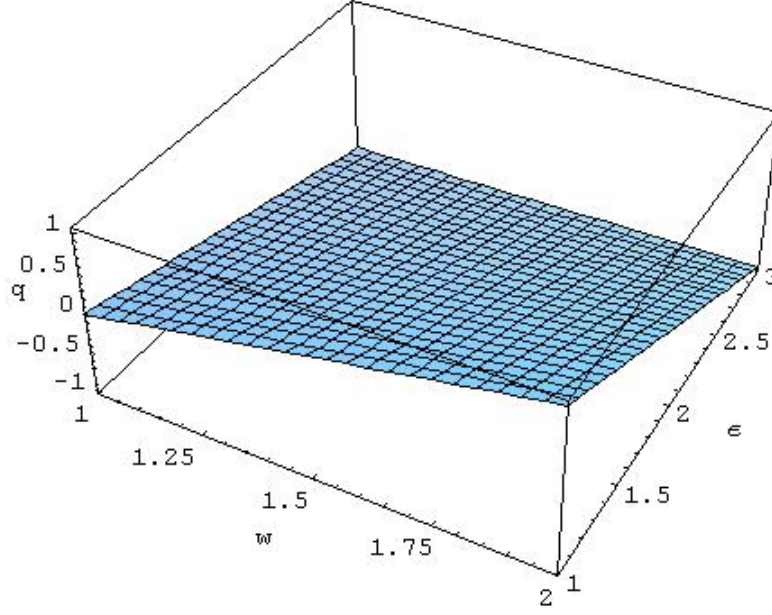
[30] A. I. Arbab, astro-ph/9810239.

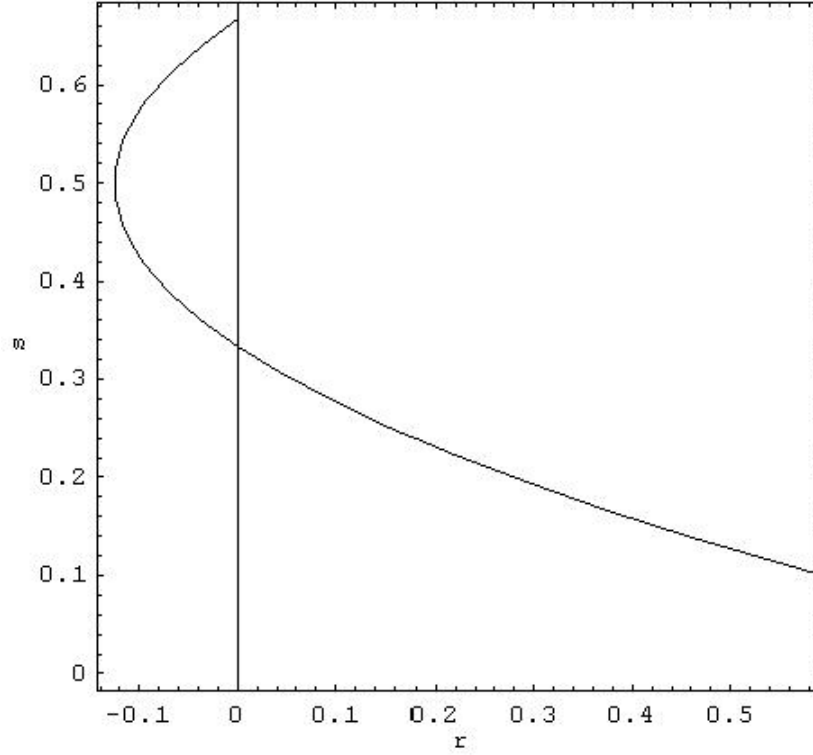FIG. 1: The deceleration parameter q is plotted for different choices of ϵ and w.

FIG. 2: The statefinder parameters are plotted for $w = 1$ and $\epsilon = 1...3$.
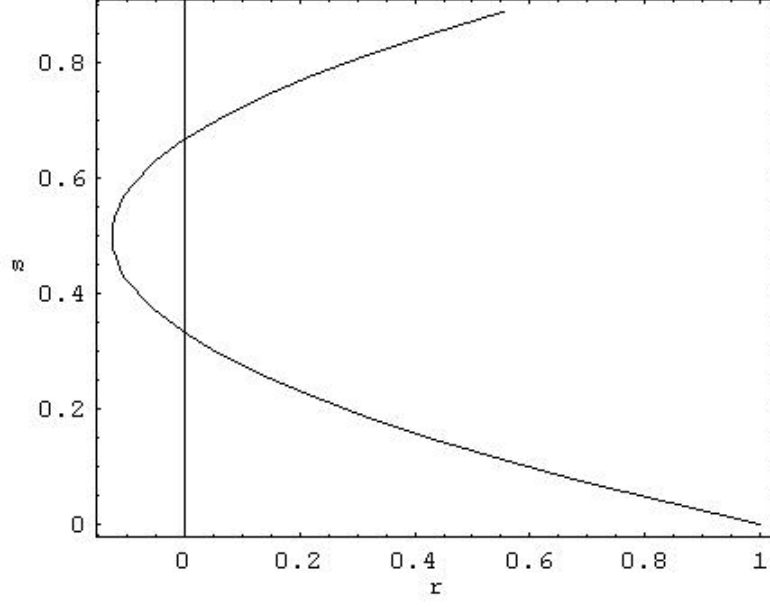
FIG. 3: The statefinder parameters are plotted for $w = 4/3$ and $\epsilon = 1...3$.
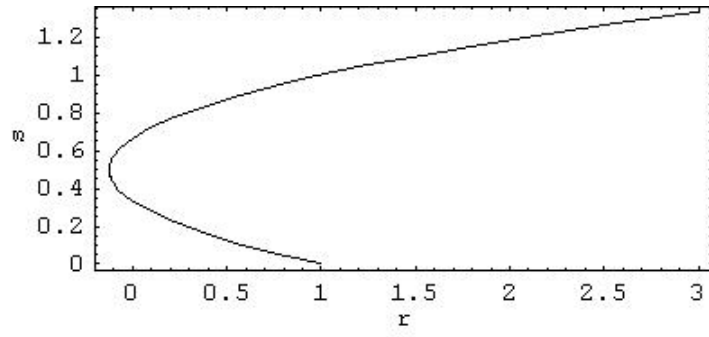

FIG. 4: The statefinder parameters are plotted for $w = 2$ and $\epsilon = 1...3$.